Geometric nature in adiabatic evolution of dark eigenstates

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In a recent Letter [Phys. Rev. Lett. **95**, 080502 (2005)], an interesting scheme was proposed to implement a type of conditional quantum phase gates with built-in fault-tolerant feature via adiabatic evolution of dark eigenstates. In this comment we elaborate the geometric nature of the gate scheme and clarify that it still belongs to a class of conventional geometric quantum computation.

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In a recent Letter [1], an interesting scheme was proposed to implement a type of conditional quantum phase gates with built-in fault-tolerant feature via adiabatic evolution of dark eigenstates. However, one of the main conclusions, the proposed conditional phase shift is neither of dynamic nor geometric origin, is found to be incorrect. Here we clarify that the phase shift acquired in the proposed gate operation is a kind of standard geometric phase (GP) defined in the literature[2], and the proposed gate scheme still belongs to a class of conventional geometric quantum computation (GQC)[3].

In fact, Aharonov and Anandan showed rigorously that once the dynamic phase is removed from the total phase shift in any cyclic evolution of a physical state, the remnant phase must be a geometric phase connected to a closed curve in the projective Hilbert space[2]. So it seems impossible that a phase shift is neither of dynamic nor geometric origin in any cyclic evolution. Moreover, a clear geometric picture demonstrated below enables us to understand the robustness of the proposed gate scheme.

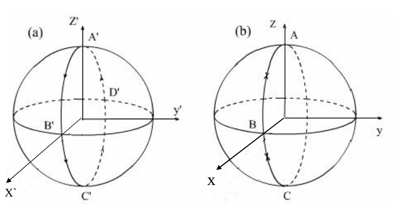

FIG. 1: The evolution paths in the Bloch sphere.

The high-dimensional space in the original system makes its geometric picture to be implicit. Nevertheless, one can decompose the whole system into two subspaces. With the same notations as in Ref. [1], the Hamiltonian described in Eqs. (1) and (8) can be truncated in the subspaces $\{|e_1\rangle|g_2\rangle|0\rangle, |g_1\rangle|e_2\rangle|0\rangle, |g_1\rangle|g_2\rangle|1\rangle\}$ and $\{|e_1\rangle|g_2'\rangle|0\rangle, |g_1\rangle|e_2'\rangle|0\rangle, |g_1\rangle|g_2'\rangle|1\rangle\}$, written as

$$H_\alpha = \begin{pmatrix} 0 & 0 & \lambda_1 \\ 0 & 0 & -\lambda_2 \\ \lambda_1^* & -\lambda_2^* & 0 \end{pmatrix}, \quad H_\alpha' = \begin{pmatrix} 0 & 0 & \lambda_1 \\ 0 & 0 & c_\alpha\lambda_3 \\ \lambda_1^* & c_\alpha\lambda_3^* & 0 \end{pmatrix},$$

respectively, where $c_1 = -1$, $c_2 = 1$, and $\alpha = 1, 2$. The dark state of the Hamiltonian H_α is given by $|D_\alpha\rangle = (\cos\theta, \sin\theta, 0)$, while is $|D_\alpha'\rangle = (-c_\alpha\cos\theta', \sin\theta', 0)$ for the Hamiltonian H_α'. We first analyze the phase shift accumulated in the evolution of the state $|D_\alpha'\rangle$ that is actually a two-level state in the subspace $\{|e_1\rangle|g_2'\rangle|0\rangle, |g_1\rangle|e_2'\rangle|0\rangle\}$, since the amplitude for the state $|g_1\rangle|g_2'\rangle|1\rangle$ is always zero. A standard scenario to look into the geometric structure of a two-level system is to study the state evolution on the Bloch sphere, where any state $|\psi\rangle$ corresponds a point at the Bloch sphere by the mapping $\mathbf{n} = \langle\psi|\sigma|\psi\rangle$ with σ being the Pauli matrix. We now examine the evolution path evolved in the gate operation proposed in Ref.[1]: θ' in $|D_1'(\theta')\rangle$ changes from 0 to $\pi/2$ driven by the Hamiltonian H_1', then θ' in $|D_2'(\theta')\rangle$ varies from $\pi/2$ to 0 governed by the Hamiltonian H_2'. By a direct calculation, we have $\mathbf{n}_\alpha' = (-c_\alpha\sin(2\theta'), 0, \cos(2\theta'))$ for the state $|D_\alpha'\rangle$. The evolution path $\mathbf{n}_\alpha'$ of the procedure in the Bloch sphere is plotted in Fig. 1a. In the first stage, the initial point is A' in Fig.1a, then the state evolves to the point C' through B'. During the second stage, the state evolves from C' to back A' through D', and thus a closed path in the Bloch sphere is formed. The solid angle enclosed by the closed path $A'B'C'D'A'$ is evidently 2π, so the GP acquired is just π. Similarly, the evolution path $\mathbf{n}_\alpha$ determined by the dark state $|D_\alpha\rangle$ is plotted as the path $ABCBA$ with a zero solid angle in Fig. 1b. Summarizing the results, we illustrate that the GPs have been acquired in the proposed gate operations described by Eqs. (10) in Ref.[1]; these GPs are essentially connected to the phase shifts during the gate operations. Therefore, the proposed gate scheme is still belong to the GQC[3], and the above analysis makes it clear that the robustness of the quantum gates stems actually from its geometric nature.

Finally, we wish to remark that the high-dimensional structure leads to a distinct advantage: the dynamic phases acquired in the gate operations are automatically zero as the states involved are dark states, which may simplify the experimental setup. In contrast, this kind of dark state can hardly be realized in a single two-level system, and thus an additional operation is normally re-

quired to cancel the dynamic phases[3]. In this sense the gate scheme proposed in Ref.[1] is quite arresting.

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